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On the Stability of the Interior Equilibrium Point of a Bioeconomic Model of Prey-Predator Interaction with Constant Harvesting and Reserve Zones

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Abstract

A modified version of a bioeconomic model for Prey-Predator interaction in polluted environment with constant harvesting strategy and reserve zone is proposed and studied. The coexistence equilibrium state has been analysed using the method of linearization and Routh-Hurwitz criteria. The analytical results revealed that the coexistence equilibrium state is locally asymptotically stable. Results of numerical experiments carried out using Matlab R2010a agree substantially with the analytical results. The implication of the results obtained, is that creation of reserve zones have significant effects on both Tilapia and Nile perch yields in the unreserve zone. Therefore, the study recommends the creation of reserve zones with migration rate of $\phi = 0.7$ for effective control of overexploitation and extinction of fish biomass in both reserve and unreserve zones.

Key words: Bioeconomic; Reserve Zone; Prey-Predator; Coexistence equilibrium point; Stability

1. Introduction

Fish has been one of the sources of protein that substantially help human beings health wise. Economically, it had been known to contribute immensely in the economic growth and development of any given country. Over the last two decades aquiculture had become an enterprise valued by the private sector, public sector (urban and rural communities) and is gaining ground especially in Nigeria. Fishing is consequentially considered as an alternative income-generating venture. Fishing in general, offers many benefits to man these includes: food, employment, business opportunities and recreational activities.

However, overfishing can reduce the fish stock or biomass of reproductive age of fish below sustainability (threshold). According to the Food and Agriculture Organization FAO (2009), in the year 2005 about 50% of the fish stock under observation experienced overexploitation or depletion. It is desirable that in the management of a renewable resource such as fish, a strategy is developed that will allow an optimum harvest rate and yet keep the populations above a sustainable level. A major current focus in fishery management is how best to ensure harvesting threshold (Brauer, and Castillo-Chavez, 2001). Clearly, the objective of the management is to devise harvesting strategies that will not drive fish species to extinction. Therefore, the notion of overexploitation and extinction of the fish populations and their various species, as well as a precautionary harvesting policy is always critical.

Meanwhile, the study of the population dynamics under the influence of harvesting coupled with Prey-predator interaction has been a subject of mathematical bioeconomics. One of the substantial reasons for studying the dynamics of the population of Tilapia (*Oreochromis niloticus*) and Nile perch (*Lates niloticus*) species in a Prey-Predator interaction coupled with harvesting, is to improve the control of overexploitation and extinction of fish species and presumably wipe off the phenomena of overexploitation and extinction of fish species. Several studies however, exploited this subject in different versions, approaches and goals which the studies intended to achieve. This study reviewed the followings (Kar, and Matsuda, 2006), (Kar, and Chakraborty, 2010), (Christou, Cameroun, and Idel 2012), (Kar, and

Matsuda, 2008), (Zhang, and Yang, 2007), (Daga, Singh, Jai, and Ujjainkar, 2012), (Kar, and Chakraborty, 2012), (Zhang, Zhang, and Bai, 2012), (Sharma, and Gupta, 2014), (Pezzey, Roberts and Urdal, 2000), and (Mayengo, Luboobi and Kuznetsov, 2014).

In this paper the model due to Mayengo, Luboobi, and Kuznetsov (2014) is modified by integrating reserve zone as an alternative strategy in fishery management, and also accounted for natural death. The habitant of these fish species is partitioned into reserve and unreserve zones. Fishing activities are restricted to the reserve zones only.

2.0 Materials and Methods.

In this section, the mathematical formulation of the compartmental model for Prey-Predator interaction with constant harvesting strategy and reserve zones is presented. The total population of Tilapia (*Oreochromis niloticus*) and Nile perch (*Lates niloticus*) species in a Prey-Predator interaction is divided into four compartments as shown in Figure 1 while the model variables and parameters are presented in Table 1 and Table 2 respectively. Hence, the compartmental model represents a biological dynamics. The economic features could not be represented in like manner, as such they have been incooperated in the model equations.

2.1 Model Description

The population of Tilapia perch in an unreserve zone $x(t)$ is naturally recruited at the rate $\gamma_1\left(1-\frac{1}{k_1}\right)$ and decreases by harvesting at the rate q_1E_1, natural death rate σ_1, death rate due to water pollution d_1 and Predation effect $\frac{\beta}{A+x}$. The Tilapia perch in an unreserve zone requires a spillover from the reserve zone at the rate ϕ_1. The population of Nile perch in an unreserve zone $y(t)$ is naturally recruited at the rate $\gamma_2\left(1-\frac{1}{k_2}\right)$, predation benefit

$\frac{\beta}{A+x}$, and spillover from the reserve zone at rate ϕ_2. Moreover, the population decrease by harvesting q_2E_2, natural death rate σ_2, death rate due to water pollution d_2. The population of Tilapia perch in reserve zone $x_R(t)$ is naturally recruited at the rate $\gamma_3\left(1-\frac{1}{k_3}\right)$ and decreases by natural death rate σ_3, and spillover from the reserve zone at rate ϕ_1. The population of Nile perch in reserve zone $y_R(t)$ is naturally recruited at the rate $\gamma_4\left(1-\frac{1}{k_4}\right)$ and decreases by natural death rate σ_4, and spillover at rate ϕ_2.

Furthermore, considering the economic feautures using the cost benefit measure for both fish species, total benefit as $\mu_1 p_1 q_1 E_1$ and $\mu_1 c_1 E_1$ as the total cost for Tilapia perch is generated; while $\mu_2 p_2 q_2 E_2$, $\mu_2 c_2 E_2$ represents total benefit and total cost for Nile perch respectively.

Table 1: Model Variables and their Description

Symbol	Descriptions
$x(t)$	Number of Tilapia perch in unreserve zone at time t
$y(t)$	Number of Nile perch in unreserve zone at time t
$x_R(t)$	Number of Tilapia perch in reserve zone at time t
$y_R(t)$	Number of Nile perch in reserve zone at time t
$E_T(t)$	Economic rent for Tilapia perch populations at time t
$E_N(t)$	Economic rent for Nile perch populations at time t

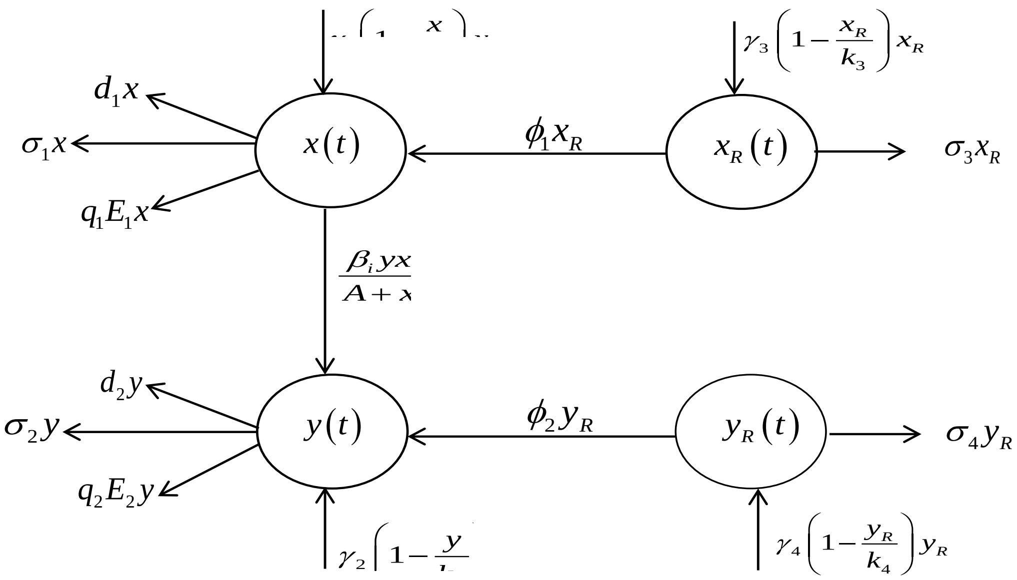
Table 2: Model Parameters and their Description

Symbol	**Descriptions**
γ_1	Intrinsic growth rate of Tilapia perch in the unreserve zone
γ_2	Intrinsic growth rate of Nile perch in the unreserve zone
γ_3	Intrinsic growth rate of Tilapia perch in the reserve zone
γ_4	Intrinsic growth rate of Nile perch in the reserve zone
k_1	Environmental carrying capacity for Tilapia perch in the unreserve zone
k_2	Environmental carrying capacity for Nile perch in the unreserve zone
k_3	Environmental carrying capacity for Tilapia perch in the reserve zone
k_4	Environmental carrying capacity for Nile perch in the reserve zone
β_1	The maximal relative increase of predation
β_2	Conversion factor from Prey to Predator
A	Saturation constant
d_1	Death rate of Tilapia perch due to water pollution
d_2	Death rate of Nile perch due to water pollution
q_1	Catchability coefficient for Tilapia perch
q_2	Catchability coefficient for Nile perch
μ_1	Stiffness parameter for Tilapia perch
μ_2	Stiffness parameter for Nile perch
p_1	Constant price per unit biomass for Tilapia perch
p_2	Constant price per unit biomass for Nile perch
c_1	Constant cost per unit biomass for Tilapia perch
c_2	Constant cost per unit biomass for Nile perch
ϕ_1	Migration rate of Tilapia perch from reserve to unreserve zone
ϕ_2	Migration rate of Nile perch from reserve to unreserve zone
σ_1	Natural death rate of Tilapia perch in unreserve zone
σ_2	Natural death rate of Nile perch in unreserve zone
σ_3	Natural death rate of Tilapia perch in reserve zone
σ_4	Natural death rate of Nile perch in reserve zone
E_1	Harvesting effort for Tilapia perch
E_2	Harvesting effort for Nile perch

2.2 Model Assumption

The following assumptions are made in the formulation of the model

i. the populations of both Preys and Predators are partitioned into reserve and unreserve zones each;
ii. migration is only one way from reserve zone to unreserve zone;
iii. water in the reserve zones is assumed pollution-free;
iv. The population in each reserve zones is assumed to be homogeneous;
v. natural death rate in reserve zone is relatively smaller than that of unreserve zone
vi. we assumed that $\gamma_2 < \gamma_1$ only in unreserve zone, but $\gamma_3 = \gamma_4$ in reserve zone



Where $i = 1, 2.$

Figure 1: Flow Diagram for the Model

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2.3 Model Equations

$$\dot{x}(t) = \gamma_1\left(1-\frac{x}{k_1}\right)x - q_1E_1x - \frac{\beta_1 yx}{A+x} - d_1x - \sigma_1 x + \phi_1 x_R \qquad (1)$$

$$\dot{y}(t) = \gamma_2\left(1-\frac{y}{k_2}\right)y - q_2E_2y + \frac{\beta_2 yx}{A+x} - d_2y - \sigma_2 y + \phi_2 y_R \qquad (2)$$

$$\dot{x}_R(t) = \gamma_3\left(1-\frac{x_R}{k_3}\right)x_R - \phi_1 x_R - \sigma_3 x_R \qquad (3)$$

$$\dot{y}_R(t) = \gamma_4\left(1-\frac{y_R}{k_4}\right)y_R - \phi_2 y_R - \sigma_4 y_R \qquad (4)$$

$$\dot{E}_T(t) = \mu_1(p_1q_1x - c_1)E_1 \qquad (5)$$

$$\dot{E}_N(t) = \mu_2(p_2q_2y - c_2)E_2 \qquad (6)$$

$$x(0)=x^0,\ y(0)=y^0,\ E_T(0)=E_T^0,\ E_N(0)=E_N^0,\ x_R(0)=x_R^0,\ y_R(0)=y_R^0,\ \ t=0$$

(7)

3.0 Results

In this section, the analytical and numerical results obtained in this work is presented below:

3.1 Coexistence Equilibrium Point of the Model

The coexistence equilibrium points of the model was obtained by equating the right hand sides of equations (1) - (6) to zero and solving the resulting system simultaneously to get

$$P^* = \left(\omega_1, \omega_2, \frac{b_1k_3}{\gamma_3}, \frac{b_2k_4}{\gamma_4}, \frac{(A+\omega_1)\left[\gamma_3(a_1k_1+\gamma_1\omega_1)+k_1k_3b_1\phi_1\right]-k_1\gamma_3\beta_1\omega_2}{k_1q_1\gamma_3(A+\omega_1)}, \frac{(A+\omega_1)\left[\gamma_4(a_2k_2+\gamma_2\omega_2)+k_2k_4b_2\phi_2\right]+k_2\gamma_4\beta_2\omega_1}{k_2q_2\gamma_4(A+\omega_1)}\right)$$

Where $a_i = (\gamma_i - d_i - \sigma_i),\ i=1,2.$ $b_i = (\gamma_j - \phi_i - \sigma_j),\ i=1,2.$ $j=3,4.$ $\omega_i = \dfrac{c_i}{p_iq_i},\ i=1,2.$

3.1.1 Conditions for existence of the model

The existence of $P_0(0,0,0,0,0,0)$ is trivial. The equilibrium points P^* exist if and only if $a_1,\ a_2,\ b_1,\ b_2 > 0$ that is if

$$\left.\begin{array}{l}\gamma_1 - d_1 - \sigma_1 > 0\\ \gamma_2 - d_2 - \sigma_2 > 0\\ \gamma_3 - \phi_1 - \sigma_3 > 0\\ \gamma_4 - \phi_2 - \sigma_4 > 0\end{array}\right\} \qquad (8)$$

The existence of the (coexistence) interior equilibrium point $P\left(x^*, y^*, x_R^*, y_R^*, E_T^*, E_N^*\right)$ is subject to conditions (8) and (9). However, conditions (8) are necessary but not sufficient conditions for the existence of the interior equilibrium point. Therefore, the sufficient conditions for the interior equilibrium point to exist are

$$\left.\begin{array}{l}\beta_1 < \dfrac{(A+\omega_1)}{k_1\gamma_3\omega_2}\left[\gamma_3\left(a_1k_1 + \gamma_1\omega_1\right) + k_1k_3\phi_1b_1\right]\\ \beta_2 > \dfrac{(A+\omega_1)}{k_2\gamma_4\omega_1}\left[\gamma_4\left(a_2k_2 + \gamma_2\omega_2\right) + k_2k_4\phi_2b_2\right]\end{array}\right\} \qquad (9)$$

Proposition 3.1.1

The interior equilibrium point $P\left(x^*, y^*, x_R^*, y_R^*, E_T^*, E_N^*\right)$ of the system (1)-(6) exists if and only if the conditions in (9) are satisfied. It follows that, the satisfaction of these conditions guarantee the coexistence of Prey, and Predator in the system alongside their economic rents.

3.2 Local stability of the Coexistence Equilibrium Point of the Model

In this section, the linearization technique and Routh-Hurwitz criteria to investigate the local stability of the coexistence equilibrium point of the model is used.

Exploring linearization technique for equations in (1) - (6) of the model gives

$$J=\begin{pmatrix} a_1-\frac{2\gamma_1}{k_1}x-q_1E_1+\frac{A\beta_1 y}{(A+x)^2} & -\frac{\beta_1 x}{(A+x)} & \phi_1 & 0 & -q_1x & 0 \\ -\frac{A\beta_2 y}{(A+x)^2} & a_2-\frac{2\gamma_2}{k_2}y-q_2E_2+\frac{\beta_2 x}{(A+x)} & 0 & \phi_2 & 0 & -q_2y \\ 0 & 0 & b_1-\frac{2x_R\gamma_3}{k_3} & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2-\frac{2y_R\gamma_4}{k_4} & 0 & 0 \\ \mu_1p_1q_1E_1 & 0 & 0 & 0 & \mu_1\left(p_1q_1x-c_1\right) & 0 \\ & \mu_2p_2q_2E_2 & 0 & 0 & 0 & \mu_2\left(p_2q_2y-c_2\right) \end{pmatrix} \qquad (10)$$

Evaluating (10) at P^*, we have (11)

$$J^*=\begin{pmatrix} \frac{(A+\omega_1)^2\left[-3\gamma_1\gamma_3\omega_1-k_1k_3b_1\phi_1\right]-k_1\gamma_3\beta_1\omega_1\omega_2}{k_1\gamma_3(A+\omega_1)^2} & -\frac{\beta_1\omega_1}{(A+\omega_1)} & \phi_1 & 0 & -q_1\omega_1 & 0 \\ -\frac{A\beta_2\omega_2}{(A+\omega_1)^2} & \frac{(A+\omega_1)\left[-3\gamma_2\gamma_4\omega_2-k_2k_4b_2\phi_2\right]+2k_2\gamma_4\beta_2\omega_1}{k_2\gamma_4(A+\omega_1)} & 0 & \phi_2 & 0 & -q_2\omega_2 \\ 0 & 0 & -b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -b_2 & 0 & 0 \\ u_1p_1\left[\frac{(A+\omega_1)(\gamma_3(a_1k_1+\gamma_1\omega_1)-k_1\gamma_3\beta_1\omega_2}{k_1\gamma_3(A+\omega_1)}\right] & 0 & 0 & 0 & 0 & 0 \\ 0 & u_2p_2\left[\frac{(A+\omega_1)(\gamma_4(a_2k_2+\gamma_2\omega_2)+k_2\gamma_4\beta_2\omega_1}{k_2\gamma_4(A+\omega_1)}\right] & 0 & 0 & 0 & 0 \end{pmatrix} \qquad (11)$$

By using Routh – Hurwitz criteria the corresponding characteristic polynomial equation of the Matrix J^* is

$$f(\lambda)=\lambda^6+m_1\lambda^5+m_2\lambda^4+m_3\lambda^3+m_4\lambda^2+m_5\lambda+m_6$$

The corresponding Hurwitz matrix is given by

$$H_6=\begin{pmatrix} m_1 & 1 & 0 & 0 & 0 & 0 \\ m_3 & m_2 & m_1 & 1 & 0 & 0 \\ m_5 & m_4 & m_3 & m_2 & m_1 & 1 \\ 0 & 0 & m_5 & m_4 & m_3 & m_1 \\ 0 & 0 & 0 & m_6 & m_5 & m_3 \\ 0 & 0 & 0 & 0 & 0 & m_6 \end{pmatrix} \qquad (12)$$

Where:

$$m_1 = (b_1 + b_2) - (s_1 + s_4)$$

$$m_2 = (b_1 + b_2)(1 - (s_1 + s_4)) - (s_1 s_4 + s_2 s_3) + q_1 \omega_1 s_5 + q_2 \omega_2 s_6$$
$$m_3 = (b_1 + b_2)\left[s_1 s_4 - s_2 s_3 - (s_1 + s_4) + q_1 \omega_1 s_5 + q_2 \omega_2 s_6\right] - (q_1 \omega_1 s_4 s_5 + q_2 \omega_2 s_1 s_6)$$
$$m_4 = (b_1 + b_2)(q_2 \omega_2 s_1 s_6 - q_1 \omega_1 s_4 s_5) + b_1 b_2 (s_1 s_4 - s_2 s_3 + q_1 \omega_1 - q_2 \omega_2 s_6) + q_1 q_2 \omega_1 \omega_2 s_5 s_6$$
$$m_5 = (b_1 + b_2)(q_1 q_2 \omega_1 \omega_2 s_5 s_6) + b_1 b_2 (q_2 \omega_2 s_1 s_6 - q_1 \omega_1 s_4 s_5)$$

$$m_6 = b_1 b_2 q_1 q_2 \omega_1 \omega_2 s_5 s_6$$

$$s_1 = \frac{(A+\omega_1)^2\left[-3\gamma_1\gamma_3\omega_1 - k_1 k_3 b_1 \phi_1\right] - k_1\gamma_3\beta_1\omega_1\omega_2}{k_1\gamma_3(A+\omega_1)^2}$$

$$s_2 = -\frac{\beta_1\omega_1}{(A+\omega_1)}$$

$$s_3 = -\frac{A\beta_2\omega_2}{(A+\omega_1)^2}$$

$$s_4 = \frac{(A+\omega_1)\left[-3\gamma_2\gamma_4\omega_2 - k_2 k_4 b_2 \phi_2\right] + 2k_2\gamma_4\beta_2\omega_1}{k_2\gamma_4(A+\omega_1)}$$

$$s_5 = u_1 p_1\left[\frac{(A+\omega_1)(\gamma_3(a_1 k_1 + \gamma_1\omega_1) - k_1\gamma_3\beta_1\omega_2}{k_1\gamma_3(A+\omega_1)}\right]$$

$$s_6 = u_2 p_2\left[\frac{(A+\omega_1)(\gamma_4(a_2 k_2 + \gamma_2\omega_2) + k_2\gamma_4\beta_2\omega_1}{k_2\gamma_4(A+\omega_1)}\right]$$

The determinant of the matrix (12) and following through the Routh-Hurwitz criteria, conditions (13) and (14) were obtained. Therefore, the necessary and sufficient condition for local stability using Routh-Hurwitz criteria resolved as given below:

$$(m_1m_4 - m_5)(m_1m_2m_3 - m_1^2 - m_1^2m_4) > m_5(m_1m_2 - m_3^2) + m_1m_1^5$$

(13)

$$m_1^3m_3m_4(m_6 + m_1m_2m_5) + 2m_1m_5(m_4m_5 + m_1m_2m_6)m_5^2 + m_3(m_3^2m_6 + m_2m_5^2) > m_5^3 + m_1m_5(m_2^2m_5 + 3m_3m_6)$$
$$+m_1m_6(m_2m_3^2 + m_1^2m_6) + m_4m_5(m_3^2 + m_1^2m_4)$$

(14)

Proposition 3.2.1

The local stability of the interior equilibrium point of the model equations in (1)-(6) is locally asymptotically stable if the conditions in (13) and (14) hold.

3.3 Numerical Simulation

In this section, the numerical simulation of the model using Runge-Kutta order four scheme is presented. The baseline values for the variables and parameters as provided in Table 3 for numerical simulations is presented. The impact of reserve zones with varied migration rate from reserve to unreserve zones is computed see table 4. In addition, the computational results of the impact of reserve zones is presented in figure 1 through 5.

Table 3: The baseline value for Variables and Parameters for the Model for Prey-Predator Interaction in Polluted Environment with Constant Harvesting Strategy and Reserve Zones

Parameter	Value	Source
γ_1	0.80	Mayengo *et al*, (2014)
γ_2	0.65	Mayengo *et al*, (2014)
γ_3	0.90	Assumed
γ_4	0.90	Assumed
k_1	600000	Mayengo *et al*, (2014)
k_2	500000	Mayengo *et al*, (2014)
k_3	600000	Mayengo *et al*, (2014)
k_4	500000	Mayengo *et al*, (2014)
β_1	0.000005	Mayengo *et al*, (2014)
β_2	0.000003	Mayengo *et al*, (2014)
A	60000	Mayengo *et al*, (2014)
d_1	0.2	Mayengo *et al*, (2014)
d_2	0.2	Mayengo *et al*, (2014)
q_1	0.000005	Mayengo *et al*, (2014)
q_2	0.000012	Mayengo *et al*, (2014)
μ_1	0.1	Mayengo *et al*, (2014)
μ_2	0.12	Mayengo *et al*, (2014)
p_1	750	Mayengo *et al*, (2014)
p_2	700	Mayengo *et al*, (2014)
c_1	500	Assumed
c_2	500	Assumed
ϕ_1	0.5	Assumed
ϕ_2	0.5	Assumed
σ_1	0.2	Assumed
σ_2	0.2	Assumed
σ_3	0.1	Assumed
σ_4	0.1	Assumed
E_1	1.20	Mayengo *et al*, (2014)
E_2	1.50	Mayengo *et al*, (2014)

Table 4: Computed Results of the Impact of Reserve Zone on Optimal Economic Rent of the Fish Species with Increased Migration Rate from Reserve to Unreserve Zone

Change of migration rate	E_T^*	E_N^*
0.5	20,000,115,591	66,754,772,177
0.6	24,000,115,591	80,105,669,726
0.7	28,000,116,697	93,456,567,276
0.8	32,000,115,591	106,807,464,825
0.9	36,000,114,485	120,158,362,374
1	40,000,116,144	133,509,254,607

Example 1:

In this example, we use baseline parameter values in table 3 with $x(0) = 40000$, $y(0) = 40000$ $x_R(0) = 40000$ and $y_R(0) = 40000$ to simulate results in figures 1 through 5

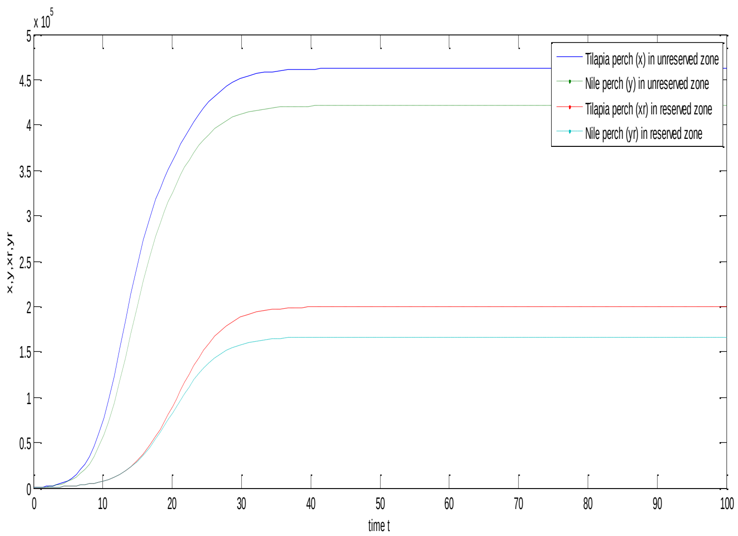

Figure 1: Simulation Results for the Impact of Reserve zone (i.e. Migration from Reserve to Unreserve Zone) Using Values of Parameters as Given in Table 3.

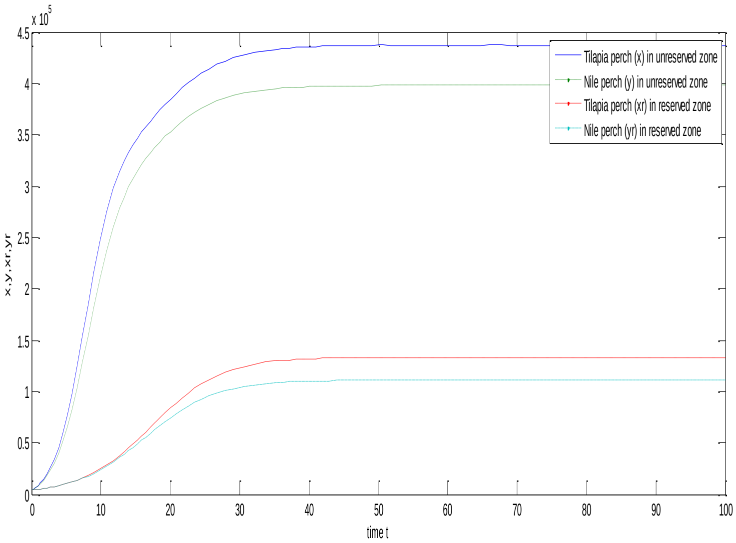

Figure 2: Simulation Results for the Impact of Reserve Zones increasing Migration Rate to 0.6 keeping all other Parameter Values as Provided in Table 3 constant

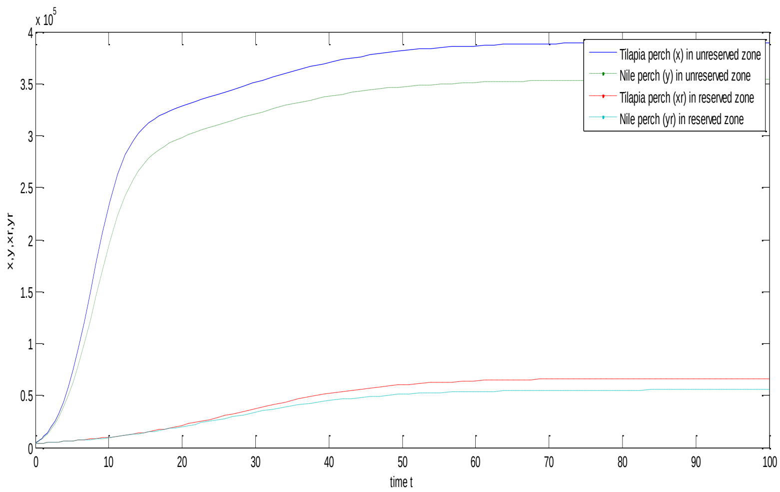

Figure 3: Simulation Results for the Impact of Reserve Zones increasing Migration Rate to 0.7 keeping all other Parameter Values as Provided in Table 3 constant

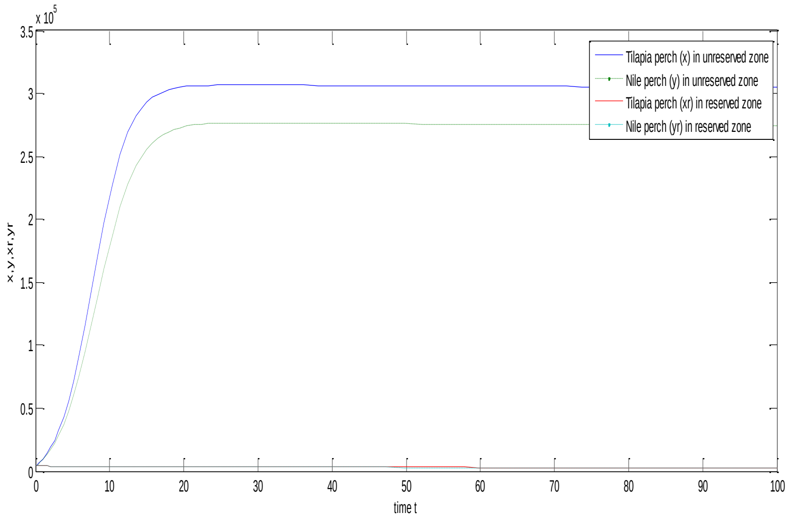

Figure 4: Simulation Results for the Impact of Reserve Zones increasing Migration Rate to 0.8 keeping all other Parameter Values as Provided in Table 3 constant

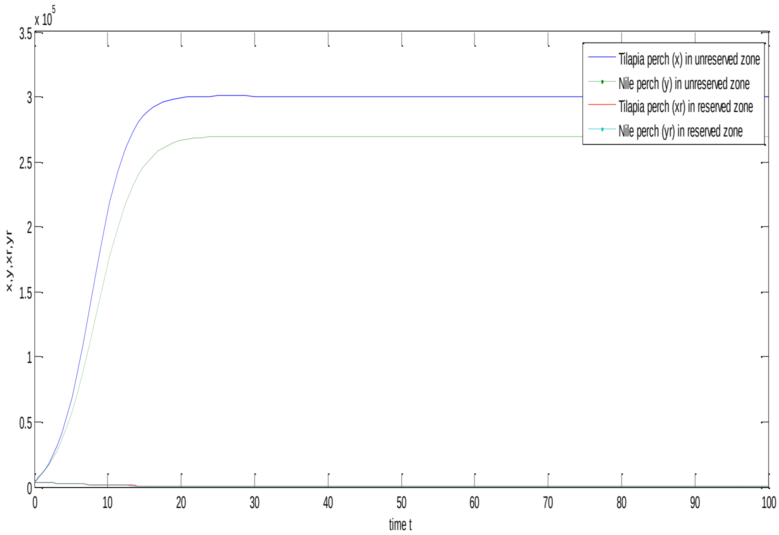

Figure 5: Simulation Results for the Impact of Reserve Zones increasing Migration Rate to 0.9 keeping all other Parameter Values as Provided in Table 3 constant

4. Discussion

In this numerical simulations, the biomass density of Tilapia and Nile perch species in the presence of reserve zones is examined. From figure 1, it is clear that biomass density of both Tilapia and Nile perch species in the unreserve zones increase more sharply (due to spill over got from the reserve zones) near their carrying capacity and settle down at their equilibrium level. But, biomass density of Tilapia and Nile perch species in reserve zones increase in snail pace near their carrying capacity and eventually settle down to their equilibrium level.

In addition, as the migration rate from reserve to unreserve zone is varied from 0.5 to 0.6, 0.7, 0.8, and 0.9 respectively as presented in figures 1 through 5. The results as plotted on the graphs indicate that, as the migration rate from reserve to unreserve zone increases, the more the reserve zones shrink consequence to periodic spill over. Similarly, as the migration rate is varied; optimal economic rent changes. Nonetheless, where the migration rate begins to affect the reserve zones negatively; optimal economic rent is at peak. This would only be achieve at $\phi = 0.7$. Migration rate beyond $\phi = 0.7$ exhibits the behavior of the model without reserve zones. Therefore, Migration rate should be adequately controlled in order not to overstretch the reserve zone; this would be achieved if steady migration rate that would not adversely affect the reserve zone is maintained.

5. Conclusion

A modified version of a bioeconomic model due to Mayengo, Luboobi, and Kuznetsov (2014) had been proposed. The study observed the long time efficiency of the reserve zone as to remedy the overexploitation and extinction of fish biomass caused by intensive harvesting, Prey-Predator interaction, and water pollution as well. Therefore, the modified model provides a better control of overexploitation and extinction of fish stock. The simulation results of the model show that, increase in migration rate beyond $\phi = 0.7$ diminishes the biomass density of Tilapia and Nile perch species in reserve zone. Thus, the ability of both the species in reserve zone to regenerate quickly is hampered and if care is not taken the species undergo an irreversible decline and cannot recover even if harvesting ceased (see figures 4 - 5). The study suggested that, migration rate at $\phi = 0.7$ should be maintained in the reserve zone as spill over to unreserve zone; so that the reserve zones would be able to adequately regenerate itself.

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